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Tugas ETS Analisis Vektor (C)

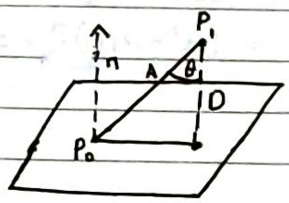
1 Buktikan jarak titik $P_0(x_0, y_0, z_0)$ ke bidang $ax+by+cz+d=0$ adalah

$$D = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$

misal, kita punya suatu titik asal P_0 yang terletak pada gambar, dan menuju suatu titik P (titik yang akan dicari jaraknya). Selain itu, tinggi bidang ke P yaitu D. Terdapat pula n yang merupakan normal bidang dan berhimpit ke P_0.

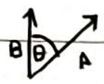
Sehingga : $A = P_0P_1 = P_1 - P_0$

$= \langle x_1-x_0,\ y_1-y_0,\ z_1-z_0 \rangle$



diperoleh : $D = \text{comp}_n A = |A|\cos\theta$

anggap



$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{b}|}$$

$$\text{proj}_b a = |\vec{a}|\cos\theta$$

Sehingga, diperoleh $D = |A|\cos\theta = \dfrac{|n.A|}{|n|}$, dengan $n = \langle a,b,c \rangle$

$$D = \frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|ax_1+by_1+cz_1-(ax_0+by_0+cz_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|ax_1+by_2+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$ (terbukti) //

Hitung jarak antara bidang $x+2y-2z=3$ dan bidang $2x+4y-4z=7$. Ambil titik awal pada $x+2y-2z=3$, dengan $y=0$ dan $z=0$. Diperoleh titik awal $(3,0,0)$. Maka jarak P ke bidang $2x+4y-4z=7$ adalah :

$$d = \frac{|2.3+4(0)+(-4)(0)+(-7)|}{\sqrt{2^2+4^2+(-4)^2}}$$

$$= \frac{|6-7|}{\sqrt{36}}$$

$$= \frac{1}{6}$$

∴ Maka jarak dari bidang $x+2y-2z=3$ dan bidang $2x+4y-4z=7$ adalah $\frac{1}{6}$ satuan jarak.

3. Dapatkan kelengkungan dari kurva $x = t$, $y = 4t^{3/2}$, $z = -t^2$ di $(1, 4, -1)$

Diketahui : $x = t$; $y = 4t\sqrt{t}$; $z = -t^2$

Ditanya : K saat P

Jawab : $r(t) = t\,\hat{i} + 4t^{3/2}\hat{j} - t^2\hat{k}$

$r'(t) = \hat{i} + 6\sqrt{t}\,\hat{j} - 2t\hat{k} \Longrightarrow r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 1 & 6\sqrt{t} & -2t \\ 0 & 3t & -2 \end{vmatrix}$

$r''(t) = \frac{3}{\sqrt{t}} \cdot \hat{j} - 2\hat{k}$

$= 3t\,k + (-12t^{1/2} + 6t^2)\,i + 2j$

$= (6t^2 - 12\sqrt{t})\,\hat{i} + 2\hat{j} + 3t\,\hat{k}$

$$K(1) = \frac{|-6\hat{i} + 2\hat{j} + 3\hat{k}|}{(\sqrt{1+36+4})^3}$$

$$= \frac{\sqrt{36+4+9}}{(\sqrt{41})^3}$$

$$= \frac{\sqrt{49}}{41\sqrt{41}}$$

$$= \frac{7}{41\sqrt{41}}$$

2. Suatu partikel bergerak mengikuti kurva dengan persamaan parametrik $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Pada waktu $t = 1$, partikel bergerak searah dengan vektor $\vec{u} = \hat{i} - 3\hat{j} + 2\hat{k}$. Tentukan laju (besarnya kecepatan) partikel waktu $t = 1$ (searah $\vec{u}$).

Diketahui : $x = 2t^2$ | Searah $\vec{u} = \hat{i} - 3\hat{j} + 2\hat{k}$

$y = t^2 - 4t$ | Saat $t = 1$

$z = 3t - 5$

Ditanya : Laju searah $\vec{u}$ saat $t = 1$

Jawab : $\vec{r}(t) = 2t^2\,\hat{i} + (t^2 - 4t)\hat{j} + (3t-5)\hat{k}$

$\vec{r}\,'(t) = 4t\,\hat{i} + (2t-4)\hat{j} + 3\hat{k}$

Agar laju searah $\vec{u}$, maka gunakan konsep proyeksi.

$$\text{Proj}_u v = \frac{u \cdot v}{|u|^2}\vec{u}$$

$$= \frac{(\hat{i} - 3\hat{j} + 2\hat{k})(4t\,\hat{i} + (2t-4)\hat{j} + 3\hat{k})}{(\sqrt{14})^2}(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$= \frac{(4t - 3(2t-4) + 6)}{14}(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$= \frac{(4t - 6t + 12 + 6)}{14}(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$= \frac{-2t + 18}{14}(\hat{i} - 3\hat{j} + 2\hat{k})$$

Saat $t = 1 \rightarrow \frac{-2 + 18}{14}(\hat{i} - 3\hat{j} + 2\hat{k})$

$$= \frac{8}{7}(\hat{i} - 3\hat{j} + 2\hat{k})$$

Besar laju = $\sqrt{\left(\frac{8}{7}\right)^2 + \left(-\frac{24}{7}\right)^2 + \left(\frac{16}{7}\right)^2}$

$= \sqrt{\frac{64+576+256}{49}}$

$= \frac{1}{7}\sqrt{896}$

$= \frac{8}{7}\sqrt{14}$ //

4. Diketahui permukaan benda $z = f(x,y)$ yang diberikan oleh persamaan parametrik:

$$\begin{cases} x = u\cos v \\ y = u\sin v \\ z = 2u \end{cases}$$

a). Tunjukkan $z = f(x,y)$ adalah kerucut.

Bentuk umum : $z^2 = x^2 + y^2$

$\Rightarrow z = k\sqrt{(x^2+y^2)}$

$2u = k\sqrt{(u^2\cos^2 v + u^2\sin^2 v)}$

$2u = k\sqrt{u^2}$

$2u = ku$

$k = 2$ //

Sehingga : $\frac{z^2}{k^2} = x^2 + y^2$ (Bentuk Umum kerucut).

b). Nyatakan permukaan benda tersebut sebagai fungsi vektor $\vec{r}(u,v)$

$\vec{r}(u,v) = u\cos v\,\hat{i} + u\sin v\,\hat{j} + 2u\,\hat{k}$

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c). Dapatkan vektor normal permukaan $z = f(x,y)$ saat $(u,v) = \left(2, \frac{\pi}{4}\right)$.

$z^2 = k^2(x^2+y^2)$

$= 4(x^2+y^2)$

$f(\varphi) = z^2 - 4x^2 - 4y^2$

$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$= (-8x)\hat{i} + (-8y)\hat{j} + 2z\hat{k}$

$x = u\cos v$

$= 2 \cdot \frac{1}{2}\sqrt{2} = \sqrt{2}$

$y = u\sin v$

$= \sqrt{2}$

$z = 2u$

$= 4$

$|\nabla\phi| = \sqrt{(-8x)^2 + (-8y)^2 + 2z^2}$

$= \sqrt{64x^2 + 64y^2 + 4z^2}$

$= \sqrt{128 + 128 + 64}$

$= \sqrt{320}$

$= 8\sqrt{5}$

$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{1}{8\sqrt{5}}\left(-8x\hat{i} - 8y\hat{j} + 2z\hat{k}\right)$

$= -\frac{\sqrt{2}}{\sqrt{5}}\hat{i} - \frac{\sqrt{2}}{\sqrt{5}}\hat{j} + \frac{1}{\sqrt{5}}\hat{k}$

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160
64
0

KKY

64
32
0

d). Dapatkan persamaan garis normal dan bidang singgung saat $(u,v) = \left(2, \frac{\pi}{4}\right)$

$$r\left(2, \frac{\pi}{4}\right) = \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + 4\hat{k}$$

▶ Persamaan Garis normal

$$R - r = k\left[\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right]$$

$$\frac{dr}{du} = \cos v\,\hat{i} + \sin v\,\hat{j} + 2k$$

$$\frac{dr}{dv} = -u \sin v\,\hat{i} + u\cos v\,\hat{j} + 0\hat{k}$$

Subs batas $\rightarrow$ $\frac{dr}{du} = \frac{1}{2}\sqrt{2}\,\hat{i} + \frac{1}{2}\sqrt{2}\,\hat{j} + 2\hat{k}$

$$\frac{dr}{dv} = -\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$$

$$\frac{dr}{du} \times \frac{dr}{dv} = \begin{vmatrix} i & j & k \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 2 \\ -\sqrt{2} & \sqrt{2} & 0 \end{vmatrix}$$

$$= 2\hat{k} - 2\sqrt{2}\,\hat{i} + 2\sqrt{2}\,\hat{j} +$$

※ Pers Garis normal

$$(R - \bar{r}) = k\left(\frac{dr}{du} \times \frac{dr}{dv}\right)$$

$$((x-\sqrt{2})\hat{i} + (y-\sqrt{2})\hat{j} + (z-4)k) = k(-2\sqrt{2}\,\hat{i} - 2\sqrt{2}\,\hat{j} + 2k)$$

$$\frac{x-\sqrt{2}}{-2\sqrt{2}} = \frac{y-\sqrt{2}}{-2\sqrt{2}} = \frac{z-4}{2}$$

※ Persamaan bidang singgung

$$(R - \bar{r}) \cdot \left[\frac{dr}{du} \times \frac{dr}{dv}\right] = 0$$

$$((x-\sqrt{2})\hat{i} + (y-\sqrt{2})\hat{j} + (z-4)k)(-2\sqrt{2}\,\hat{i} - 2\sqrt{2}\,\hat{j} + 2k) = 0$$

$$-2\sqrt{2}(x-\sqrt{2}) - 2\sqrt{2}(y-\sqrt{2}) + 2(z-4) = 0$$

$$-2\sqrt{2}x + 4 - 2\sqrt{2}y + 4 + 2z - 8 = 0$$

$$-2\sqrt{2}x - 2\sqrt{2}y + 2z = 0$$

$$2x + 2y - 2\sqrt{2}z = 0$$

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